Hume on Intuitive and demonstrative Inference

This paper is divided into four sections. The first section deals with Hume's attempt to resolve a dilemma concerning the objects of intuitive and demonstrative inference. In the second section I try to show that his resolution of the dilemma is hard to reconcile with his phenomenalist doctrine of the origin of ideas. In the third section I examine the meaning of "intuition" in Hume. Finally, in the fourth section I compare his view of mathematical inference with his view of causal inference, a kind of inference to which he tends to assimilate the former.

I

What are the objects of intuitive and demonstrative inference in Hume supposed to be? In the Treatise they are co-extensive with four kinds of philosophical relations—resemblance, contrariety, degrees in quality and proportions in quantity or numbers. The distinguishing characteristic of the group, according to Hume, is that they all depend entirely on the ideas which we compare together. The example he gives is that of a triangle whose three angles are equal to two right ones. And this presumably is at the same time an example of a relation of proportion in quantity or numbers. Needless to say, what cries out for explanation here, apart from the notion of comparison with which we shall deal later, is the phrase "depend entirely upon the ideas." Hume himself does not provide the explanation. Consequently, a certain amount of speculation becomes inevitable. One fairly plausible although, as we shall see, not entirely satisfactory interpretation of the phrase in question would associate it with a thesis concerning the reducibility of relations between exemplars of properties to relations between the properties themselves. Thus it may
be argued that the equality holding between the three angles of a triangle and two right angles is reducible to an equality holding between certain of their properties. The properties would be the 180° that each set of angles exemplifies.

Indirect evidence for this interpretation is provided by Hume's account of the relations of contiguity and distance, species presumably of relations of time and place, which do not, according to him, depend upon the ideas which are compared together. Indeed, these relations may be changed by "an alteration of their place, without any change on the objects themselves or on their ideas." The place" in its turn "depends on a hundred different accidents which cannot be foreseen by the mind." Leaving aside Hume's failure or refusal to draw a clear distinction here between ideas and objects which probably has to do with his belated realization that only the latter have places and are contiguous to or distant from one another, one notes his insistence that an object's place is not an essential property of the object. For his use of the word "accident" in the above-quoted passage has very little to do with its ordinary use and everything to do with its logical one where it is contrasted with "essential." Furthermore, the sentence immediately preceding the one containing that passage gives some indication of what Hume takes its logical use to be: A determinate property is an accidental one if loss of that property does not affect the object's identity or, as he would put it, the object itself. Nor is the accidental nature of the property anything but reinforced by its dependence logical or otherwise upon other accidental properties which like all such properties require an appeal to observation and induction in order to determine whether they are exemplified by that object. The loss of an essential property, on the other hand, would affect the object's identity because such properties are contained in the very definition of the object. And it suffices to understand the definition - Hume's foreseeing by the
mind comes in here - in order to determine that the property is exemplified by the object.

How does this account provide indirect evidence for our interpretation of Hume? The answer is relatively straightforward. According to that interpretation, relations which depend entirely on the ideas which we compare together are those where relations between exemplars of properties are reducible to relations between the properties themselves. And such an interpretation is implied by his exclusion of contiguity and distance from the class of such relations on the grounds that relations between places to which they are presumably reducible - what else would justify the introduction of place in that context? - involve a property, place, which is not essential to the relata. Reduction to a relation between essential properties is, however, reduction to a relation between properties which is all that our interpretation requires.

But a reduction to a relation between accidental properties like places is equally a reduction to properties. As a result, our interpretation wherein the distinction between accidental and essential properties plays no role would allow contiguity and distance as examples of relations of time and place to be relations which depend entirely on the ideas which we compare together. And this, of course, is quite contrary to Hume's intentions. His intentions, unfortunately, are not altogether consistent. Take degrees in quality, for example, which Hume does include in the class of relations which depend entirely on the ideas which we compare together. None of the qualities which he mentions in this regard - colour, taste, heat and cold - would seem to be essential properties anymore than place is. Indeed, they constitute part of the traditional list of secondary properties. Moreover, Hume himself mentions them in this regard. Such properties, however, were thought to exist in the mind as opposed to the object, an opinion vindicated in his view by a proper use of causal reasoning even though it runs counter to the promptings of the imagination. But, surely, in order
for something to be an essential property of an object it must be a property of it simpliciter. Nor is such a requirement in any way affected by Hume's insistence that the subjectivity of the secondary properties implies that of the primary ones. For one does not show that some properties are essential by showing that none is.

What we have said about degrees in quality could be applied with obvious modifications to resemblance and contrariety, two other relations included in the class of relations which depend entirely on the ideas which we compare together. Contrariety, moreover, presents special difficulties of its own. For Hume maintains that no objects are contrary to one another save existence and non-existence. And this seems to be an elliptical way he has of saying that no objects are contrary to one another save where the one has the property of existing and the other does not. Thus, he can conclude in his prolonged discussion of causality that no real objects are contrary to one another. But even if existence and non-existence are properties - a view that Hume seems concerned to deny elsewhere - they are not necessary properties of all the objects to which they are ascribed. Indeed, the general tenor of Hume's philosophy which gains its clearest expression in the Enquiry is to deny that they are necessary properties of any objects at all. Such a denial, however, renders existence and non-existence indiscernible in this regard from place. Why, as a result, should contrariety between objects be reducible to contrariety between them if contiguity and distance between objects are not reducible to contiguity and distance between places?

Hume is faced, then, with a dilemma. He can accept our minimal interpretation of relations which depend entirely on the ideas which we compare together. This will allow him to include degrees in quality, resemblance and contrariety among such relations but, as already indicated, he will by the same token be compelled to include at least some relations of time and place, namely contiguity and distance. Moreover, the possibility cannot be ruled out in advance that identity and causality,
the two remaining philosophical relations, will have to be included as well. The inclusion of causality, however, would result in the assimilation of the corresponding inferences to those of the intuitive and demonstrative kind. For there would be no relevant difference on which to base a distinction between the two kinds of inference if causality itself turned out to be a relation which depends entirely on the ideas which we compare together. If, on the other hand, Hume rejects our minimal interpretation of such relations in favour of one requiring the related properties on which the relations between objects depend to be essential to the objects involved, then only proportions in quantity or number would seem unqualifiedly to meet such a requirement.

That the dilemma is genuine and not one that we have foisted upon Hume is indicated by his later attempt to resolve it in favour of its second horn. For in the Enquiry the distinction between relations of ideas and matters of fact parallels the one drawn earlier in the Treatise between relations which depend entirely on the ideas which we compare together and those which do not. And both of the examples he gives of relations of ideas are mathematical in nature, the one drawn from geometry, the other from arithmetic. But these along with algebra constitute the domain of proportions of quantity or number of the Treatise. Hume, moreover, makes it perfectly clear that in his view this domain is now the unique source of objects of intuitive and demonstrative inference. Degrees in quality, resemblance and contrariety are no longer thought capable of providing such objects.

If, however, Hume limits the number of sources of objects of intuitive and demonstrative inference, he extends the range of the sole remaining one to possible objects. Propositions expressing a relation of ideas, he informs us, are "discoverable by the mere operation of thought without dependence on what is anywhere existent in the universe." Consequently, "though there never were a circle or triangle in nature, the truths demonstrated by Euclid would for ever retain their certainty and evidence."
Such an extension is, moreover, implied by the requirement that the related properties of a relation of ideas be essential to the objects whose relations they in turn ground. For essential properties, as indicated earlier, are in Hume's view contained in the very definitions of objects. And relations between these properties can ground relations between the objects defined by their means whether or not anything exists corresponding to the definitions.

II

There is nonetheless a problem here generated by Hume's commitment to phenomenalism. "All the perceptions of the human mind," he holds, "resolve themselves into two distinct kinds which I shall call Impressions and Ideas." And by ideas he means here images of impressions. Furthermore, it is "impossible for us so much as to conceive or form an idea of any thing specifically different from ideas and impressions." But would relations of images continue to hold if nothing corresponded to them in rerum natura? The answer as far as Hume is concerned must be in the negative as far as simple images are concerned. For every simple image is parasitic upon a simple impression to which it corresponds. As a result, a relation of simple images to which nothing corresponded in rerum natura would be a relation without relata. In other words, it would be an impossibility. But the absence of simple images would guarantee the absence of the complex ones which are constructed out of the former. Thus a relation of complex images would equally be an impossibility and for the very same reason.

III

Nor is the identification of ideas and images limited to those contexts where Hume is delineating the elements of his philosophy. On the contrary, it manages to insinuate itself even into the discussion of the objects of intuitive
and demonstrative inference. Thus, when he says in the Treatise that degrees in quality, resemblance and contrariety are more properly regarded as objects of intuition than demonstration, it is not clear whether he means by intuition a form of immediate inference or, rather, sense perception. The former interpretation is appropriate, if at all, to relations between properties, the latter to relations between images and even more impressions themselves. Indeed, that Hume himself was not sure of what he meant is graphically confirmed by the following sentence, "When any objects resemble each other, the resemblance will at first strike the eye, or rather the mind; and seldom requires a second examination." The key phrase, of course, is "strike the eye or rather the mind." 25

The same confusion, moreover, infects his discussion of proportions in quantity or number which, as we have seen, he retains under a different name in the Enquiry. Hume believes that there are cases where equalities between angles can be perceived. 26 Indeed, one suspects that his example already mentioned of a triangle whose three angles are equal to two right ones is meant to be just such a case. How else is one to explain his assertion that such a relation of equality "is invariable, as long as our idea remains the same?" 27 For an image of a triangle can change but a triangle itself cannot, unless, of course, it is legitimate to identify the two. The only other alternative would be to attribute to Hume something akin to the Cartesian view on which mathematical structures are created by God and can be changed by Him. 28 There is no evidence however, that he even entertained such an extraordinary view.

If Hume believes that there are cases where equalities between angles can be perceived, there are, nonetheless, in his view other cases where they cannot be perceived. He mentions in this regard the angles of a chiliagon which are equal to 1996 right angles. 29 And it is natural to assume that it is the numbers of angles which are of decisive importance here. But what of numbers themselves as opposed to the comparative sizes of
angles? Are the comparative sizes of the former in any cases perceivable? Hume seems prepared to answer that question in the affirmative. Thus he holds that we "might at one view observe a superiority or inferiority betwixt any numbers, or figures: especially where the difference is very great and remarkable." Even if it is true, however, that a substantial difference between numbers is in some metaphorical sense observable, it is not literally observable. Moreover, it is not in the slightest degree plausible to identify numbers with images as it may be so to identify angles. For, while there may be a correlation between the size of an image of an angle and the size of the corresponding angle on the assumption that images of angles have sizes, there is no such correlation when a number is substituted for an angle. The size of my image of the number five, for example, has nothing to do with the place that number occupies in the series of natural numbers. Nor is it likely that Hume thought otherwise. It seems rather that he has fallen victim to his own ambiguous use of "intuition."

Since there is a genuine ambiguity here, any attempt to eliminate it in favour of one interpretation of the term in question will be somewhat arbitrary. Failure to appreciate this fact has led one group of Hume scholars to hold that he never meant to speak of properties of objects actual or possible at all but only of images. As a result, they are forced to adopt a tortured interpretation of that passage in the Enquiry already quoted which clearly seems to refer to such properties. Thus Zabeeh takes the assertion, "Though there never were a circle or triangle in nature, the truths demonstrated by Euclid would for ever retain their certainty and evidence," and insists that in making this very assertion Hume did not mean to include Euclid among those for whom these figures were absent and so, presumably, unperceived. But, surely, if there were no such figures in nature, Euclid would suffer the same deprivation as the rest of us. Indeed, if he did not, that would be proof positive that these figures did exist in nature. For their enjoyment of
this status is not only compatible with but is implied by their manifesting themselves to at least one person.

If, however, Zabeeh is forced to adopt a tortured interpretation of a key passage in the *Enquiry* in order to render Hume's position consistent, Kant, for his part, simply ignores whole sections of the *Treatise* and significant passages of the *Enquiry* when he asserts that Hume had too much insight to base the axioms of pure mathematics on sense perception.\(^3\) As a result, one wonders if he would have revised his estimation of his illustrious predecessor in this regard upon learning of geometry that "Its first principles are still drawn from the general appearance of the objects...."\(^4\) And this is not an isolated passage.

Despite its somewhat arbitrary nature an unambiguous interpretation of "intuition" is nonetheless desirable. Which interpretation should it be? It seems to me that it should be the one on which intuition is intellectual as opposed to sensuous. For, if Kant's judgment of Hume on this score has to be qualified, the former was surely right in his refusal to base the axioms of pure mathematics on sense perception. And it is a disservice to Hume in my view to emphasize an interpretation which inextricably commits him to falsehood when there is another one available which possibly commits him to truth. This does not mean, of course, that we have to or should follow Kant and certain positivists in attributing to Hume the view that the propositions of mathematics are analytic.\(^5\) Indeed, such an attribution seems to be based on no more than the assumption that he had logical contradiction in mind when he held the negation of a proposition expressing a relation of ideas to be contradictory. Such an assumption, however, fails to explain the importance that he attaches to conceivability in this regard. Conceivability, as Pap's research tends to show, is in Hume's view a sufficient condition for non-contradictoriness.\(^6\) Moreover, as we have already had occasion to note, the phenomenalist in him leads him to identify ideas, the objects, presumably of our conceiving, with images. Thus
his notion of contradiction tends to be closely tied to the image-forming capacities of human beings—a question of interest to the psychologist but hardly the logician.

IV

But what of the distinction mentioned above between ideas and our conceiving of them? Nowhere as far as I know does Hume provide an analysis of what it is to be an idea which would permit him to distinguish as Descartes did between an idea as an object of mental act of a conceiving and an idea as the act of conceiving itself. One could, needless to say, make the distinction and omit any analysis which would justify it. Indeed, when the occasion suited him this is exactly what Hume seems to have done. Thus, in his attempt to explain in what a belief in a matter of fact consists he is forced to supplement the idea believed with the manner of its being conceived. And he calls this "an act of the mind." The same presumably would hold for impressions where they are relevant to the matter of fact believed. Hume, moreover, allows for transitions between these mental acts. Indeed, the comparison of ideas in which he takes intuitive and demonstrative inference to consist and of which we promised to speak earlier is supposed to resemble the determination of the mind to make one such transition, namely, the one constituting causal inference, as the following passage makes clear:

Thus as the necessity which makes two times two equal to four, or three angles of a triangle equal to two right ones, lies only in the act of understanding, by which we consider and compare these ideas; in like manner the necessity or power, which unites causes and effects, lies in the determination of the mind to pass from the one to the other.

For, if the resemblance in question is to hold, there must be "acts of understanding" in both cases. Two of these in the one case presumably would sustain the transition
the mind is determined to make mentioned in the passage.

How many kinds of necessity does Hume countenance? The passage itself does not allow us to answer that question. For the comparison of ideas and the determination of the mind to make the transition between them could be identified each in its turn with a different kind of necessity. Fortunately, he informs us a little later that there is but one kind of necessity. Admittedly, this assertion is made in anticipation of an attempt to distinguish between moral and physical necessity but there is no reason to believe that Hume had that distinction alone in mind in making the assertion. He insists, moreover, that only a determination of the mind to make a transition and nothing less is relevant to necessity. Thus the comparison of ideas in which he takes intuitive and demonstrative inference to consist must in the final analysis be a species of such a determination as opposed to merely resembling one.

There are nonetheless problems here. If both causal inference on the one hand and intuitive and demonstrative inference on the other hand consist in the determination in question, we should expect Hume to provide us with the same explanation of how they are produced in both cases. For one of the rules he lays down by which to judge of causes and effects is that "the same cause always produces the same effect, and the same effect never arises but from the same cause." He further describes this rule as "the source of most of our philosophical reasonings." The fact remains, however, that far from providing the same explanation Hume provides no explanation at all of the determination of the mind to make the transition between mental acts in which mathematical necessity is supposed to reside. In the case of the determination of the mind to make the transition where causal necessity or power is supposed to be found on the other hand regularities holding between objects which are spatially contiguous and where the one succeeds the other provide the explanation. Hume's analysis of causal inference, in other words, is itself a causal analysis.
And it is none the worse for that. But the materials for such an analysis do not seem to be available when it is a question of intuitive and demonstrative inference. As a result, we seem to be faced with the absurdity of a determination of the mind in which there is nothing to determine it.

Nor is that all. For what could necessity equate with power have to do with the $180^\circ$ that the three angles of a triangle and two right angles each exemplify? Very little it would seem. Why then should one conclude that a determination of the mind to make a transition is relevant here at all? In the case of the causal relation on the other hand such a determination of the mind provides faute de mieux the source of that power which in Hume's view we all mistakenly attribute to the relation itself. That he describes this source of power as an impression of reflection after having so brilliantly demonstrated that there is no impression of power results from an unfortunate adherence to his official view that there are no mental acts and, therefore, no transitions between them but only mental objects - a view from which, as we have seen, he is prepared to diverge when the occasion suits him. Would that the occasion had suited him in this instance! It might have permitted an unsympathetic critic like Prichard to appreciate Hume's achievement in this area instead of berating him constantly for his lack of logical rigour. The fact remains, nonetheless, that since power has nothing to do with mathematical necessity a determination of the mind could never be the source of such necessity.

What then is the source of such necessity? Hume, it seems to me, has no choice but to seek it in the relations of ideas themselves. Indeed, despite the importance he attaches to a determination of the mind in this connection more often than not he actually does seek it in such relations. Thus, one of the basic assumptions of his search for causal power is that if it were to be found in the causal relation itself as opposed to a
determination of the mind the relation would be transformed into an object of demonstrative inference. And, as we have seen, in the *Enquiry* an object of that kind turns out to be part of the domain of relations of ideas. But such a transformation would be successful only if necessity was a characteristic of relations of ideas themselves as opposed to a determination of the mind. Otherwise we should have done no more than exchange a determination of the mind which can at least be explained for one which cannot.

The asymmetry of causal and mathematical necessity with regard to their source does, however, expose Hume to an *ad hominem* argument in favour of a kind of existence for causal necessity which is also independent of a determination of the mind. For, if the absence of the corresponding impression does not deprive mathematical necessity of such an existence, why should it do so in the case of causal necessity? And Hume, even if he had not identified the two, cannot give us any reason to believe that the corresponding impression is not absent in the case of mathematical necessity.

Should he, then, have granted in the interest of consistency that causal necessity is independent of a determination of the mind? It seems to me that we should give a negative answer to this question. It has already been noted that Hume's relations of ideas to which mathematical necessity pertains are hard to render consistent with his commitment to phenomenalism. Nor will anything be changed in this regard by giving an account of causal necessity that further violates the commitment in question. Indeed, this second violation would have the further disadvantage of being gratuitous in a way that the first one is not. For the latter is, as we have suggested, required for an adequate account of mathematics. Hume, on the other hand, provides us with an analysis of the causal relation, the so-called uniformity analysis which, whatever faults it may have, shows that we can do
1. Hume contrasts philosophical with natural relations. The essential difference between the two is supposed to be that the former require a comparison of the relata while the latter require only that we associate the relata in accordance with them. The distinction is obscured, however, by Hume's tendency to assimilate comparisons to the kind of association involved according to him in causal inference. We shall have something to say about this tendency later. For Hume's description of the distinction in question see A Treatise of Human Nature, ed. L. A. Selby-Bigge (Oxford, 1888), pp. 13-14. Hereafter cited as Treatise.

2. Treatise, p. 69.

3. Treatise, p. 69. Although Hume speaks of relations of time and place, he does not seem to have regarded place itself as such a relation. Indeed, he virtually identifies an object's place with its extension. See Section V, Part IV of Book 1 of the Treatise entitled "Of the immortality of the soul." See particularly pp. 234-39.

4. Treatise, p. 69.

5. It may also have something to do with his contention that the vulgar view - and we are all vulgar most of the time in Hume's view - makes no distinction between perceptions and physical objects. See Section II, Part IV of Book 1 of the Treatise entitled "Of scepticism with regard to the senses."

6. Thus Hume can argue that because the property of being the shortest way between two points is not - at least according to him - an essential but merely an accidental property of a right line it does not constitute a definition of such a line. Presumably, if it were essential, it would constitute such a definition. His way of describing a property as accidental here is to say that it is considered by accident. Nor is this surprising if we reflect upon other instances of Hume's propensity to look for the source of logical properties in the workings of the human mind. Indeed, in the fourth section of this
It is interesting to find a twentieth-century logician like Louis Couturat making the same connection between definability and foreseeability by the mind. "En effet, de ce que les concepts mathématiques sont fabriqués a priori et n'existent que par leur définition même, il résulte que l'esprit sait d'avance tout ce qu'il y a mis, et ne peut plus porter sur eux que des jugements analytiques." We shall have more to say about Hume and analyticity later. See Louis Couturat, "La Philosophie des Mathématiques de Kant," Revue de Métaphysique et de Morale, 12th year, 1904, p. 333. Also quoted by Henri Lauener in Hume und Kant (Bern, 1969), p. 44.


10. See Section IV, Part II of Book I of the *Treatise* entitled "Of the modern philosophy"


14. See Section VI, Part II of Book I of the *Treatise* entitled "Of the idea of existence and of external existence."


22. *Treatise*, p. 67. One wonders whether the first idea mentioned in this passage is an image. If it is not, it constitutes an exception to the rule that ideas are images. If it is, the implicit identification in the passage of concepts and ideas becomes questionable. For, even if I cannot form an image of anything specifically different from images and
impressions, this does not provide a reason for believing that I could not have the concept of something specifically different.

23. The celebrated missing shade is, of course, an exception to the rule. See Treatise, pp. 5-6. See also Enquiry, pp. 20-21.


27. Treatise, p. 70.


29. Treatise, p. 72.

30. Treatise, p. 70.


34. Treatise, p. 71.


36. Arthur Pap, Semantics and Necessary Truth (New Haven and London, 1958) pp. 75-76. Nor is there any substantial difference in this regard between the Treatise and the Enquiry. For in the latter as in the former non-contradictoriness and conceivability go hand in hand. Thus I see no grounds for Noxon's contention that a concept of logical possibility is to be found in the Enquiry as opposed to the Treatise. And as for the related contention that the Enquiry deals with types of propositions whereas the Treatise deals with a theory of relations, Hume's own words suggest that he would take this to be a false dichotomy "That the square of the hypothenuse is equal to the square of the two sides, is a proposition which expresses a relation between these figures." Enquiry, p. 25. See Noxon, p. 163.

38. Treatise, p. 627.


40. Treatise, p. 171.

41. This is something that escapes Heinrich Hasse despite the importance he attaches to Hume's comparison of logical and causal necessity. See Das Problem der Gültigkeit in der Philosophie David Humes (Münch, 1920), pp. 126–27. It does not escape Norman Kemp-Smith. See _The Philosophy of David Hume_ (London, 1941), pp. 252–53.

42. Treatise, p. 173.

43. Treatise, p. 173.

44. Treatise, pp. 164–65.


